

Hylogenesis: A Unified Origin for Baryonic Visible Matter and Antibaryonic Dark Matter

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We present a novel mechanism for generating both the baryon and dark matter densities of the Universe. A new Dirac fermion X carrying a conserved baryon number charge couples to the Standard Model quarks as well as a GeV-scale hidden sector. CP-violating decays of X , produced non-thermally in low-temperature reheating, sequester antibaryon number in the hidden sector, thereby leaving a baryon excess in the visible sector. The antibaryonic hidden states are stable dark matter. A spectacular signature of this mechanism is the baryon-destroying inelastic scattering of dark matter that can annihilate baryons at appreciable rates relevant for nucleon decay searches.

I. Introduction: Precision cosmological measurements indicate that a fraction $\Omega_b \simeq 0.046$ of the energy content of the Universe consists of baryonic matter, while $\Omega_d \simeq 0.23$ is made up of dark matter (DM) [1]. Unfortunately, our present understanding of elementary particles and interactions, the Standard Model (SM), cannot account for the abundance of either observed component of non-relativistic particles.

In this *Letter* we propose a unified mechanism, hylogenesis¹, to generate the baryon asymmetry and the dark matter density simultaneously. The SM is extended to include a new hidden sector of states with masses near a GeV and very weak couplings to the SM. Such sectors arise in many well-motivated theories of physics beyond the SM, and have received much attention within the contexts dark matter models [2], and high luminosity, low-energy precision measurements [3].

The main idea underlying our mechanism is that some of the particles in the hidden sector are charged under a generalization of the global baryon number (B) symmetry of the SM. This symmetry is not violated by any of the relevant interactions in our model. Instead, equal and opposite baryon asymmetries are created in the visible and hidden sectors, and the Universe has zero total B . These asymmetries are generated when (i) the TeV-scale states X_1 and its antiparticle \bar{X}_1 (carrying equal and opposite B charge) are generated non-thermally in the early Universe (e.g., during reheating), and (ii) X_1 decays into visible and hidden baryonic states. The X_1 decays violate quark baryon number and CP, and occur away from equilibrium. Both the visible and hidden baryons are stable due to a combination of kinematics and symmetries. The relic density of the hidden baryons is set by their asymmetry, and they make up the dark matter of the Universe. We compute the baryon and dark matter densities within a concrete model realizing

this mechanism in Section II.

A potentially spectacular signature of our model is that rare processes can transfer baryon number from the hidden to the visible sector. Effectively, antibaryonic dark matter states can annihilate baryons in the visible sector through inelastic scattering. These events mimic nucleon decay into a meson and a neutrino, but are distinguishable from standard nucleon decay by the kinematics of the meson. In Section III, we discuss this signature in more detail, along with its implications for direct detection and astrophysical systems.

We note that our scenario shares some elements with Refs. [4–11], but involves a different production mechanism and unique phenomenological consequences.

II. Genesis of Baryons and DM: In our model, the hidden sector consists of two massive Dirac fermions X_a ($a = 1, 2$, with masses $m_{X_2} > m_{X_1} \gtrsim \text{TeV}$), a Dirac fermion Y , and a complex scalar Φ (with masses $m_Y \sim m_\Phi \sim \text{GeV}$). These fields couple through the “neutron portal” ($XU^c D^c D^c$) and a Yukawa interaction:

$$-\mathcal{L} \supset \frac{\lambda_a}{M^2} \bar{X}_a P_R d \bar{u}^c P_R d + \zeta_a \bar{X}_a Y^c \Phi^* + \text{h.c.} \quad (1)$$

Many variations on these operators exist, corresponding to different combinations of quark flavors and spinor contractions. With this set of interactions one can define a generalized global baryon number symmetry that is conserved, with charges $B_X = -(B_Y + B_\Phi) = 1$. The proton, Y , and Φ are stable due to their B and gauge charges if their masses satisfy

$$|m_Y - m_\Phi| < m_p + m_e < m_Y + m_\Phi. \quad (2)$$

Y and Φ are the “hidden antibaryons” that comprise the dark matter. Furthermore, there exists a physical CP-violating phase $\arg(\lambda_1^* \lambda_2 \zeta_1 \zeta_2^*)$ that cannot be removed through phase redefinitions of the fields.

We also introduce a hidden $U(1)'$ gauge symmetry under which Y and Φ have opposite charges $\pm e'$, while X_a is neutral. We assume this symmetry is spontaneously broken at the GeV scale, and has a kinetic mixing with SM

¹ From Greek, *hyle* “primordial matter” + *genesis* “origin.”

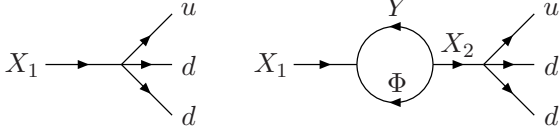


FIG. 1: Tree-level and one-loop graphs for decay $X_1 \rightarrow udd$.

hypercharge $U(1)_Y$ via the coupling $-\frac{\kappa}{2}B_{\mu\nu}Z'_{\mu\nu}$, where $B_{\mu\nu}$ and $Z'_{\mu\nu}$ are the $U(1)_Y$ and $U(1)'$ field strength tensors. At energies well below the electroweak scale the effect of this mixing is primarily to generate a vector coupling of the massive Z' gauge boson to SM particles with strength $-c_W\kappa Q_{em}e$. The GeV-scale Z' masses we consider here can be consistent with observations for $10^{-6} \lesssim \kappa \lesssim 10^{-2}$ [3].

Baryogenesis begins when a non-thermal, CP-symmetric population of X_1 and \bar{X}_1 is produced in the early Universe. These states decay through $X_1 \rightarrow udd$ or $X_1 \rightarrow \bar{Y}\Phi^*$ (and their conjugates). An asymmetry between the partial widths for $X_1 \rightarrow udd$ and $\bar{X}_1 \rightarrow \bar{u}\bar{d}\bar{d}$ arises from interference between the diagrams shown in Fig. 1, and is characterized by

$$\epsilon = \frac{1}{2\Gamma_{X_1}} [\Gamma(X_1 \rightarrow udd) - \Gamma(\bar{X}_1 \rightarrow \bar{u}\bar{d}\bar{d})] \quad (3)$$

$$\simeq \frac{m_{X_1}^5 \text{Im}[\lambda_1^* \lambda_2 \zeta_1 \zeta_2^*]}{256\pi^3 |\zeta_1|^2 M^4 m_{X_2}},$$

where we have assumed that the total decay rate Γ_{X_1} is dominated by $X_1 \rightarrow \bar{Y}\Phi^*$ over the three-quark mode, and that $m_{X_2} \gg m_{X_1}$. For $\epsilon \neq 0$, X_1 decays generate a baryon asymmetry in the visible sector, and by CPT an equal and opposite baryon asymmetry in the hidden sector. These asymmetries can be “frozen in” by the weakness of the coupling between both sectors.

We model the non-thermal production of X_1 as a reheating process after a period where the energy content of the Universe was dominated by the coherent oscillations of a scalar field φ . This field could be the inflaton, or it could be a moduli field arising from an underlying theory with supersymmetry [12] or a compactification of string theory [13]. As φ oscillates, it decays to visible and hidden sector states reheating these two sectors. We suppose that a fraction of the φ energy density ρ_φ is converted into X_1, \bar{X}_1 states, while the remainder goes into visible and hidden sector radiation which quickly thermalizes due to gauge interactions.

The dynamics of hylogenesis and reheating are governed by the Boltzmann equations

$$\frac{d}{dt}(a^3 \rho_\varphi) = -\Gamma_\varphi a^3 \rho_\varphi, \quad (4a)$$

$$\frac{d}{dt}(a^3 s) = +\Gamma_\varphi a^3 \rho_\varphi / T, \quad (4b)$$

$$\frac{d}{dt}(a^3 n_B) = \epsilon \mathcal{N}_X \Gamma_\varphi a^3 \rho_\varphi / m_\varphi \quad (4c)$$

with φ mass m_φ and decay rate Γ_φ . $s \equiv s_{\text{HS}} + s_{\text{SM}} = (2\pi^2/45)g_s T^3$ is the total entropy density of SM and HS

states (assumed in kinetic equilibrium at temperature T with an effective number of entropy degrees of freedom $g_s(T)$), and n_B is the baryon number density in the visible sector (i.e. quarks). The scale factor $a(t)$ is determined by the Friedmann equation $H^2 \equiv (\dot{a}/a)^2 = (8\pi G/3)(\rho_\varphi + \rho_r)$, where $\rho_r \equiv (\pi^2/30)gT^4$ is the total radiation density and $g(T)$ is the effective number of degrees of freedom. \mathcal{N}_X is the average number of X_1 states produced per φ decay.

Eq. (4a) describes the depletion of the oscillating field energy due to redshifting and direct φ decays and has the simple solution $\rho_\varphi \propto e^{-\Gamma_\varphi t} a^{-3}$, while Eq. (4b) gives the rate of entropy production due to decays and describes the reheating of the Universe. We adopt the convention that reheating occurs at temperature T_{RH} , defined when $\rho_r(T_{RH}) = \rho_\varphi(T_{RH})$. This occurs near the characteristic decay time $t \simeq \Gamma_\varphi^{-1}$, where the total decay width Γ_φ takes the form [9, 13] $\Gamma_\varphi = m_\varphi^3/(4\pi\Lambda^2)$. Here, Λ is a large energy scale corresponding to the underlying ultraviolet dynamics. For example, $\Lambda \sim M_{\text{Pl}} = 2.43 \times 10^{18}$ GeV for many moduli in string theory or supergravity. At reheating, the radiation temperature is approximately [9]

$$T_{RH} \simeq 5 \text{ MeV} \left(\frac{10}{g}\right)^{1/4} \left(\frac{M_{\text{Pl}}}{\Lambda}\right) \left(\frac{m_\varphi}{100 \text{ TeV}}\right)^{3/2}. \quad (5)$$

We require $T_{RH} \gtrsim 5 \text{ MeV}$ to maintain successful nucleosynthesis.

Eq. (4c) determines the comoving density of visible baryons. The remnant of the intermediate X_1 stage appears in the right-hand-side of Eq. (4c). The factor ϵ encodes the X_1 decay asymmetry. In writing Eq. (4) we implicitly take $m_{X_1} \gg T$ and $\Gamma_{X_1} \gg \Gamma_\varphi, H$. The former condition implies inverse decays and scattering reactions that could wash out the asymmetry, such as $\bar{u}X_1 \rightarrow dd$, are suppressed by Boltzmann factors of $e^{-m_{X_1}/T}$, while the latter condition is satisfied for $|\zeta_1| \gg m_\varphi^2/(m_{X_1}\Lambda)$. The hidden-visible baryon asymmetry can also be washed out by $Y\Phi \rightarrow 3\bar{q}$ scattering. A sufficient condition for this washout process to be ineffective is

$$T_{RH} \lesssim (2 \text{ GeV}) \left(\sum_{a,b} \frac{\lambda_a \lambda_b^* \zeta_a^* \zeta_b \text{ TeV}^6}{M^4 m_{X_a} m_{X_b}} \right)^{-1/5}. \quad (6)$$

The allowed T_{RH} increases roughly linearly with the mass scale $(M^4 m_{X_{1,2}}^2)^{1/6}$.

The resulting baryon asymmetry today is given by

$$n_B \equiv n_B/s = \frac{\epsilon \mathcal{N}_X T_{RH}}{m_\varphi} f(m_\varphi \Gamma_\varphi). \quad (7)$$

Assuming that reheating occurs instantaneously, one can show analytically that $f = 3/4$. A numerical solution to Eqs. (4) reveals $f \simeq 1.2$, with less than 10% variation over a wide range of $(m_\varphi, \Gamma_\varphi)$. Larger values of T_{RH} (larger m_φ for fixed Λ) allow for greater production of baryons.

For the parameter values $m_\varphi = 2000$ TeV, $\Lambda = M_{\text{Pl}}$, $\mathcal{N}_X = 1$, we find $T_{RH} \simeq 400$ MeV and $\eta_B/\epsilon \simeq 2.5 \times 10^{-7}$. The observed value of the baryon asymmetry is obtained for $\text{Im}[\lambda_1^* \lambda_2 \zeta_2 / \zeta_1] m_{X_1}^5 / (M^4 m_{X_2}) \sim 3$. Smaller values of ϵ and m_φ are viable for $\Lambda < M_{\text{Pl}}$.

We have implicitly assumed that the Z' maintains kinetic equilibrium between the SM and hidden sectors. This will occur if $\Gamma_{Z'} \langle 1/\gamma \rangle > H$, where γ is a relativistic time dilation factor, which implies [14]

$$\kappa > 1.5 \times 10^{-8} \left(\frac{g}{10} \right)^{1/2} \left(\frac{m_{Z'}}{\text{GeV}} \right)^{-1} \left(\frac{T}{\text{GeV}} \right)^{3/2}, \quad (8)$$

provided $T_{RH} > m_{Z'}$. After baryogenesis, the CP-symmetric densities of hidden states are depleted very efficiently through annihilation $Y\bar{Y} \rightarrow Z'Z'$ and $\Phi\Phi^* \rightarrow Z'Z'$ provided $m_{Z'} < m_Y, m_\Phi$, with the Z' decaying later to SM states by mixing with the photon. The cross-section for $Y\bar{Y} \rightarrow Z'Z'$ is given by [14]

$$\begin{aligned} \langle \sigma v \rangle &= \frac{e'^4}{16\pi m_Y^2} \sqrt{1 - m_{Z'}^2/m_Y^2} \\ &\simeq (1.6 \times 10^{-25} \text{ cm}^3/\text{s}) \left(\frac{e'}{0.05} \right)^4 \left(\frac{3 \text{ GeV}}{m_Y} \right)^2. \end{aligned} \quad (9)$$

Annihilation of $\Phi^*\Phi$ is given by a similar expression. These cross sections are much larger than what is needed to obtain the correct DM abundance by ordinary thermal freeze-out, and all of the non-asymmetric DM density will be eliminated up to an exponentially small remainder [15]. Note that the annihilation process may occur later than is typical for thermal freeze-out for $T_{RH} \lesssim m_{Y,\Phi}/20$, but even in this case the remaining non-asymmetric density will be negligibly small [16].

The role of the hidden Z' in our model is to ensure the thermalization and symmetric annihilation of Y and Φ . A more minimal alternative is to couple Φ to the SM Higgs boson h via the operator $\xi |h|^2 |\Phi|^2$. For $\xi \gtrsim 10^{-3}$, this interaction, together with $YX\Phi$ and $|\Phi|^4$, appears to be sufficient for both thermalization and symmetric annihilation.

The residual CP-asymmetric density of Y, Φ is not eliminated and makes up the DM [17]. The relic number density is fixed by total baryon number conservation: $n_Y = n_\Phi = n_B$. Thus the ratio between the energy densities of DM and visible baryons is

$$\Omega_d/\Omega_b = (m_Y + m_\Phi)/m_p. \quad (10)$$

Present cosmological observations imply $\Omega_d/\Omega_b = 4.97 \pm 0.28$ [1], which corresponds to a range $4.4 \text{ GeV} \lesssim m_Y + m_\Phi \lesssim 4.9 \text{ GeV}$, or $1.7 \text{ GeV} \lesssim m_Y, m_\Phi \lesssim 2.9 \text{ GeV}$ when combined with the constraint $|m_Y - m_\Phi| < m_p + m_e$.

III. Dark Matter Signatures: A novel signature of this mechanism is that DM can annihilate nucleons through inelastic scattering processes of the form

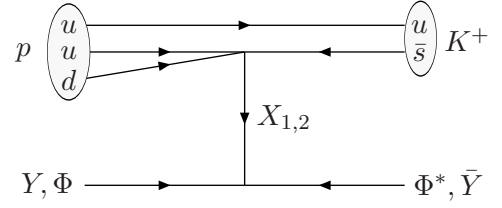


FIG. 2: Diagram for induced nucleon decay processes $pY \rightarrow K^+\Phi^*$ and $p\Phi \rightarrow K^+\bar{Y}$.

$YN \rightarrow \Phi^*M$ and $\Phi N \rightarrow \bar{Y}M$ mediated by $X_{1,2}$, where N is a nucleon and M is a meson (Fig. 2). We call this process induced nucleon decay (IND). IND mimics standard nucleon decay $N \rightarrow M\nu$, but with different kinematics of the daughter meson, summarized in Table I. For down-scattering processes, where the mass of the initial DM state is greater than the final DM state, the meson momentum p_M from IND can be much greater than in nucleon decay. The quoted range of p_M corresponds to the range of allowed masses (m_Y, m_Φ) consistent with Eqs. (2, 10). For fixed masses, p_M is monochromatic, with negligible broadening from the local DM halo velocity. (We also note a related study considering lepton-number violating inelastic DM-nucleon scattering [18].)

To estimate the rate of IND we consider the specific operator $(\lambda_a/M^2)(\bar{u}^c d)_R(\bar{X}s)_R$ that mediates $\Phi p \rightarrow \bar{Y}K^+$, illustrated in Fig. 2. Treating the Φ and Y states as spectators, the hadronic matrix element can be estimated from the value computed for the $p \rightarrow K^+\nu$ decay through the corresponding three-quark operator [19]. We find that the sum of the IND scattering rates $p\Phi \rightarrow K^+\bar{Y}$ and $pY \rightarrow K^+\Phi^*$ is given by

$$(\sigma v)_{IND} = \mathcal{C} (10^{-39} \text{ cm}^3/\text{s}) \left| \sum_a \frac{\text{TeV}^3}{m_a M^2 / \lambda_a^* \zeta_a} \right|^2 \quad (11)$$

where $0.5 < \mathcal{C} < 1.6$, depending on $m_{\Phi,Y}$ within the allowed range. We expect IND modes from other operators to be roughly comparable. This estimate, which relies on a chiral perturbation theory expansion that is expected to be poorly convergent for $p_M \sim 1$ GeV, is approximate at best.

An effective proton lifetime τ_p can be defined as the inverse IND scattering rate per target nucleon, $\tau_p^{-1} = n_{DM}(\sigma v)_{IND}$. With a local DM density of $0.3 \text{ GeV}/\text{cm}^3$, $(\sigma v)_{IND} = 10^{-39} \text{ cm}^3/\text{s}$ corresponds to a lifetime of $\tau_p \simeq 10^{32} \text{ yr}$. This is similar to the current lifetime bound

Decay mode	p_M^{SND} (MeV)	p_M^{IND} (MeV)
$N \rightarrow \pi$	460	800 - 1400
$N \rightarrow K$	340	680 - 1360
$N \rightarrow \eta$	310	650 - 1340

TABLE I: Daughter meson $M \in \{\pi, K, \eta\}$ momentum p_M for standard nucleon decay (SND) and down-scattering IND.

on $p \rightarrow K^+ \nu$ of $2.3 \times 10^{33} \text{ yr}$ [20]. However, existing nucleon decay bounds may not directly apply to IND due to the non-standard meson kinematics [21], and additional suppression can arise from the second factor in Eq. (11).

There is also a direct detection signal in our model due to the hidden Z' : Y and Φ can scatter elastically off protons. The effective scattering cross-section per nucleon for either Y or Φ is spin-independent and given by

$$\sigma_0^{SI} = (5 \times 10^{-39} \text{ cm}^2) \left(\frac{2Z}{A} \right)^2 \left(\frac{\mu_N}{\text{GeV}} \right)^2 \times \left(\frac{e'}{0.05} \right)^2 \left(\frac{\kappa}{10^{-5}} \right)^2 \left(\frac{0.1 \text{ GeV}}{m_{Z'}} \right)^4, \quad (12)$$

where μ_N is the DM-nucleon reduced mass. For a DM mass of 2.9 GeV, this is slightly below the current best limit from CRESST [22]. The effective nucleon cross-section will be much lower if the hidden vector is replaced by a Φ -Higgs coupling.

Annihilation of DM can generate energetic particles that destroy the products of nucleosynthesis [23], create a neutrino flux [24], and modify the properties of the cosmic microwave background [25]. In our scenario there is almost no direct DM annihilation after freeze-out, but similar effects can arise from IND scattering. However, the rate for this scattering given in Eq. (11) is many orders of magnitude less than the corresponding limits on DM annihilation cross-sections [23, 25], and thus we expect that IND will have no visible effect in the cosmological setting.

The effects of IND can become important in astrophysical systems with very large densities of nucleons such as neutron stars and white dwarfs. In both cases, we find that the rate of IND in the stellar core typically becomes large enough that it reaches a steady state with the rate of DM capture through elastic scattering [26, 27]. This has the effect of heating the stellar interior in much the same way as DM annihilation. Nucleons are also destroyed, but the number is only a tiny fraction of the total. Current observations of white dwarfs constrain stellar heating by DM or IND, but the bounds depend on the local DM densities which are not known precisely [28, 29]. These bounds are much weaker than for monopole catalysis of nucleon decay [30] since the anti-DM product of IND is unable to destroy any more nucleons.

IV. Conclusions: We have presented a novel mechanism to generate dark matter and baryon densities simultaneously. Decays of a massive X_1 state split baryon number between SM quarks and antibaryons in a hidden sector. These hidden antibaryons constitute the dark matter. An important signature of this mechanism is the destruction of baryons by the scattering of hidden dark matter.

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- [1] E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 330 (2009).
 - [2] M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
 - [3] J. D. Bjorken, R. Essig, P. Schuster and N. Toro, *Phys. Rev. D* **80**, 075018 (2009).
 - [4] R. Kitano and I. Low, *Phys. Rev. D* **71**, 023510 (2005).
 - [5] G. R. Farrar and G. Zaharijas, *Phys. Rev. Lett.* **96**, 041302 (2006).
 - [6] K. Agashe and G. Servant, *JCAP* **0502**, 002 (2005).
 - [7] R. Kitano and I. Low, [hep-ph/0503112].
 - [8] H. An, S. L. Chen, R. N. Mohapatra and Y. Zhang, *JHEP* **1003**, 124 (2010).
 - [9] R. Allahverdi, B. Dutta and K. Sinha, [1005.2804 [hep-ph]].
 - [10] J. Shelton and K. M. Zurek, [1008.1997 [hep-ph]].
 - [11] N. Haba and S. Matsumoto, arXiv:1008.2487 [hep-ph].
 - [12] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, *Phys. Lett. B* **131**, 59 (1983).
 - [13] B. S. Acharya, G. Kane and E. Kuflik, [1006.3272 [hep-ph]].
 - [14] M. Pospelov, A. Ritz and M. B. Voloshin, *Phys. Lett. B* **662**, 53 (2008).
 - [15] K. Griest and D. Seckel, *Nucl. Phys. B* **283**, 681 (1987) [Erratum-ibid. B **296**, 1034 (1988)].
 - [16] B. S. Acharya, G. Kane, S. Watson and P. Kumar, *Phys. Rev. D* **80**, 083529 (2009).
 - [17] D. E. Kaplan, M. A. Luty and K. M. Zurek, *Phys. Rev. D* **79**, 115016 (2009).
 - [18] J. Kile and A. Soni, *Phys. Rev. D* **80**, 115017 (2009).
 - [19] S. Aoki *et al.* [JLQCD Collaboration], *Phys. Rev. D* **62**, 014506 (2000).
 - [20] K. Kobayashi *et al.* [Super-Kamiokande Collaboration], *Phys. Rev. D* **72**, 052007 (2005).
 - [21] H. Davoudiasl *et al.* “Induced Nucleon Decay,” in preparation.
 - [22] G. Angloher *et al.*, *Astropart. Phys.* **18**, 43 (2002).
 - [23] J. Hisano, M. Kawasaki, K. Kohri, T. Moroi and K. Nakayama, *Phys. Rev. D* **79**, 083522 (2009).
 - [24] J. F. Beacom, N. F. Bell and G. D. Mack, *Phys. Rev. Lett.* **99**, 231301 (2007).
 - [25] T. R. Slatyer, N. Padmanabhan and D. P. Finkbeiner, *Phys. Rev. D* **80**, 043526 (2009).
 - [26] C. Kouvaris, *Phys. Rev. D* **77**, 023006 (2008).
 - [27] G. Bertone and M. Fairbairn, *Phys. Rev. D* **77**, 043515 (2008).
 - [28] M. McCullough and M. Fairbairn, *Phys. Rev. D* **81**, 083520 (2010).
 - [29] D. Hooper, D. Spolyar, A. Vallinotto and N. Y. Gnedin, *Phys. Rev. D* **81**, 103531 (2010).
 - [30] K. Freese, M. S. Turner and D. N. Schramm, *Phys. Rev. Lett.* **51**, 1625 (1983).